A surface one inch square is thrown at random upon a surface one foot square, but so as always to lie wholly upon the larger surface. Find the mean value of *the sum of the distances* of the vertices of the smaller surface, from *any* vertex of the larger surface.

19. Proposed by H. W. DRAUGHON, Clinton, Louisiana.

From one corner of a square field, a boy runs in a random direction, with a random uniform velocity. The greatest distance the boy can run in one minute is equal to the diagonal of the field. What is the probability that the boy will be in the field at the end of one minute?

Solutions to these problems should be received on or before December 1st.

MISCELLANEOUS.

Conducted by J M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

4. Proposed by J K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania.

I have two grindstones, each $\frac{1}{2}$ inch thick. One is 6 in. and the other $4\frac{1}{2}$ in. in diameter, the aperature at center of each being $1\frac{1}{2}$ in. If when in motion they are continually tangent to each other, and $\frac{1}{2}$ cu. in. is ground off the larger wheel and $\frac{1}{4}$ cu. in. off the smaller in the first hour, how must their speed be increased so that the same amount per hour may be ground off each wheel until one is worn out? If in the first hour the larger wheel makes a revolutions, and the smaller b, how many must each make in each succeeding hour?

II. Solution by P. H. PHILBRICK, C. E., Lake Charles, Louisiana.

The diagonal of the aperature is, $\sqrt{4.5}=2.1213$. Vol. of larger stone outside of the circle circumscribing the aperature is, $\frac{1}{2}\frac{\pi}{4}(36-4.5)=12.37$ and the same for the smaller stone is, $\frac{1}{2}\frac{\pi}{4}(20.25-4.5)=6.185$. The stones will therefore wear out at the same time and in $12.37\div\frac{1}{2}=24.74$ hours.

The side face of larger stone including the aperature$=\frac{\pi}{4}36=28.27$; and the available area$=\frac{\pi}{4}(36-4.5)=24.74$. One inch area is therefore worn off the side face of the larger stone per hour and one half of an inch off the face of the smaller stone. Hence, including the aperature, the side face of the larger stone at the end of the first hour is 27.27.

At end of second hour 26.27.

At end of third hour 25.25.

etc.,etc.

At the end of the $24\frac{1}{2}$ hour is 4.27.

The areas of the circumscribing squares at end of successive hours differ by $1\div\frac{\pi}{4}=\frac{4}{\pi}$. Hence letting D=the diameter of stone at the beginning; D_1, D_2, etc., the same at the end of the 1st, 2d hours, etc. we have,

$$D=\sqrt{36}\quad=6$$

$$D_1=\sqrt{36-\frac{4}{\pi}}=5.8929$$

$$D_2=\sqrt{36-\frac{8}{\pi}}=5.7839$$

$$D_3=\sqrt{36-\frac{12}{\pi}}=5.6728$$

etc., etc.

$$\text{Finally, } D_{24}=\sqrt{36-\frac{96}{\pi}}=2.3329.$$

Hence diameter worn off 1st hour$=6-5.8929=0.1071$,
and the 2d hour$=5.8929-5.7839=0.1090$,
and the 3rd hour$=5.7839-5.6728=0.1111$,
etc. etc.

The diameter worn off after the end of 24 hours is, $2.3329-2.1213=0.2116$.

Now $\frac{1}{2}\frac{\pi}{4}[(2.3329)^2-(2.1263)^2]=0.37$ of a cubic inch as it ought.

In 24 hours, $24\times\frac{1}{2}=12$ inches are ground off, leaving $12.37-12=0.37$ of an inch, as just found, and this will consume $0.37\div\frac{1}{2}=.74$ of an hour, or 24.74 hours in all.

In the same way the diameters worn off the smaller stone are found.

The number of revolutions per hour are precisely as the diameters ground off and are therefore shown above.

[Prof. Philbrick solves for square apertaure and in part under a slightly different interpretation from Prof. Hume, whose solution was previously published. We publish above for comparison. H. W. Draughon's solution agreed with Prof. Hume's, except that it was in general terms.—Editor.]

6, **Proposed by H. C. WHITAKER, B S., M. E., Professor of Mathematics, Manual Training School, Philadelphia. Pennsylvania.**

Two men wish to buy a grindstone 42 inches in diameter and one foot thick at the center. To what thickness at the outer edge should the stone uniformly taper from the center so that each man may grind off 18 inches of the diameter and both have equal shares, the central six inches of the diameter being waste?

III. Solution by the PROPOSER.

[Refer to fig. used on pg. 174 of the May number]

Let equation of line generating the lateral surface of the stone be $x=2a-3by$.

$$\text{Then Vol}=\int 4\pi xy\,dy=4\pi\int(2ay-3by^2)dy=4\pi(ay^2-by^3)$$

Taking first the limits to be 21 and 12 and then 12 and 3 and equating